# Quantitative Skills in AP Environmental Science

This chapter focuses on some of the quantitative skills that are important in your AP Environmental Science course, in addition to all of the skills covered in chapters 1, 2, and 3. The skills in this chapter are not all of the skills that you will learn, practice, and apply during the year, but these are the skills you will most likely encounter as part of your laboratory investigations or classroom experiences, and potentially on the AP Environmental Science Exam.

Primary Productivity in Ecosystems

The **primary productivity** of an ecosystem is the rate at which organic compounds are stored. Primary producers convert the energy from sunlight into organic compounds by the process of photosynthesis:

$$6CO_2 + 6H_2O \longrightarrow C_6H_{12}O_6 + 6O_2$$
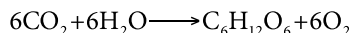
Gross primary productivity (GPP) is the rate at which plants convert solar energy into chemical energy (organic compounds). **Net primary productivity (NPP)** represents the energy in the organic compounds left over for consumers after the primary producers meet their own needs through cellular respiration:

$$NPP = GPP - R$$

where R represents the energy that is used for cellular respiration.

There are three ways to measure primary production:

- Amount of carbon dioxide used
- Rate of oxygen production
- Rate of glucose production

The efficiency of photosynthesis can be calculated from the net primary productivity using the following equation:

$$Efficiency = (NPP / Insolation\ Energy) \times 100$$

where the insolation energy is the average daily incident solar energy.

Example

On a maize farm in Ethiopia, the insolation energy is 5,654,875 *calories/m²/day*. The gross productivity of the rice is 0.02742 *g/cm²/day*, and 35% of the energy is lost due to respiration. There are 4000 calories in 1 gram of maize. Calculate the net primary productivity of the maize and the efficiency of photosynthesis of the maize.

$$\frac{0.02742\,g}{cm^2\,/\,day} \times \frac{100\,cm^2}{1m^2} \times \frac{4000\,cal}{1g} = 10{,}968\,cal\,/\,m^2\,/\,day$$

$$NPP = GPP - R$$

$$NPP = 10{,}968\ cal\,/\,m^2\,/\,day - 3{,}839\,cal\,/\,m^2\,/\,day$$

$$NPP = 7{,}129\,cal\,/\,m^2\,/\,day$$

$$Efficiency = (NPP\,/\,Insolation\ Energy) \times 100$$

$$Efficiency = \frac{7129\,cal\,/\,m^2\,/\,day}{5{,}654{,}875\,cal\,/\,m^2\,/\,day} \times 100$$

$$Efficiency = 0.0013 \times 100$$

$$Efficiency = 0.13\%$$

If you need more information, the following tutorial can help to further explain these concepts:

Khan Academy: Energy flow & primary productivity

Survivorship Curves

Survivorship curves are graphs that show the relative survival rates of a cohort of organisms at different ages. There are three types of survivorship curves (also illustrated in figure 5.1):

- Type I survivorship curve: Shows high survival rates throughout a species life span (flat curve at younger ages), with a high rate of death at advanced ages. Organisms that demonstrate this pattern include animals that have few offspring but invest high levels of parental care, such as large mammals.
- Type II survivorship curve: Shows a constant decline of survivorship in a species' life span. Organisms with this kind of survivorship include annual plants, small mammals, invertebrates, and lizards.
- Type III survivorship curve: Shows low survivorship in early age, with few members reaching old age. These organisms generally have large numbers of offspring and the parental investment is low. Examples include marine organisms and some plants.
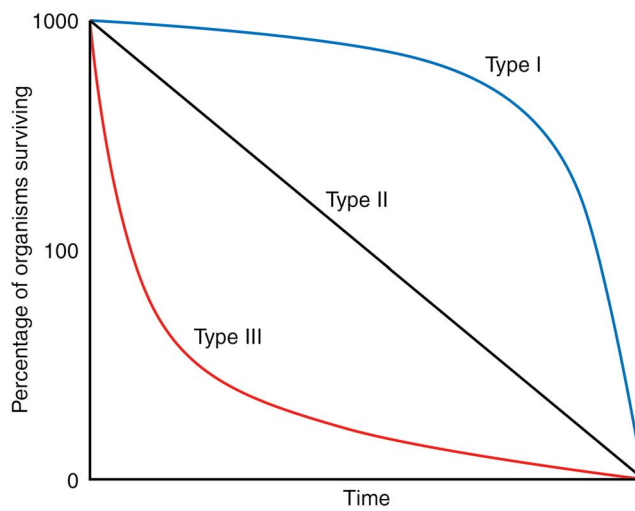
Figure 5.1: Survivorship Curves

Survivorship curves differ for *K*-selected versus *r*-selected species, with *K*-selected species typically following a type I or type II curve, and *r*-selected species following a type III curve.

If you need more information, the following tutorial can help to further explain these concepts:

Khan Academy: Human population growth

Populations

Population Growth

Populations can experience unlimited growth when resources are abundant, but this is rarely the case in nature.

The change in the size of a population over a given amount of time mostly depends on the number of births and the number of deaths that occur within it. This can be written mathematically as

$$\frac{\Delta N}{\Delta t} = B - D$$

where ΔN is the change in the population, Δt is the change in time, B is the number of births, and D is the number of deaths.

CHAPTER 5

*Quantitative
Skills in AP
Environmental
Science*

Populations

In a given population, the per capita rate of increase, *r*, is equal to

$$r = \frac{B-D}{N}$$

The equation for the change in population size can now be written as

$$\frac{\Delta N}{\Delta t} = rN$$

Two basic models of population growth are the exponential growth model and the logistic growth model:

- Exponential growth model: A population increases in size when *r* is constant and greater than zero:

$$\frac{\Delta N}{\Delta t} = r_{max}N$$

  where r_{max} is the maximum per capita rate of increase and *N* is the size of the population. In other words, the population (usually a small one) increases by a fixed rate.

- Logistic growth model: A population increases in size until *r* approaches zero as the size of the population reaches its carrying capacity:

$$\frac{\Delta N}{\Delta t} = r_{max}N\frac{(K-N)}{K}$$

where *K* is the carrying capacity of the population. The carrying capacity is the maximum population size that can be supported by a particular environment at a certain point in time without disruption to the habitat.

Exponential growth example

Suppose a population of 100 bacteria is experiencing exponential growth: the current birth rate is 22 bacteria per day and the current death rate is 15 individuals per day. Calculate the growth of this population over 30 days.

First, let's calculate the per capita rate of increase:

$$r = \frac{B-D}{N}$$
$$r = \frac{22-15}{100}$$
$$r = 0.7$$

Now, let's calculate the growth rate for day one:

$$\frac{\Delta N}{\Delta t} = rN$$
$$\frac{\Delta N}{\Delta t} = (0.7)(100)$$
$$\frac{\Delta N}{\Delta t} = 70$$

CHAPTER 5

*Quantitative
Skills in AP
Environmental
Science*

Populations

This means that 70 bacteria are added the first day. So, for the second day, we start with 170 bacteria. Using this same equation, we can calculate the size of the population for the next day:
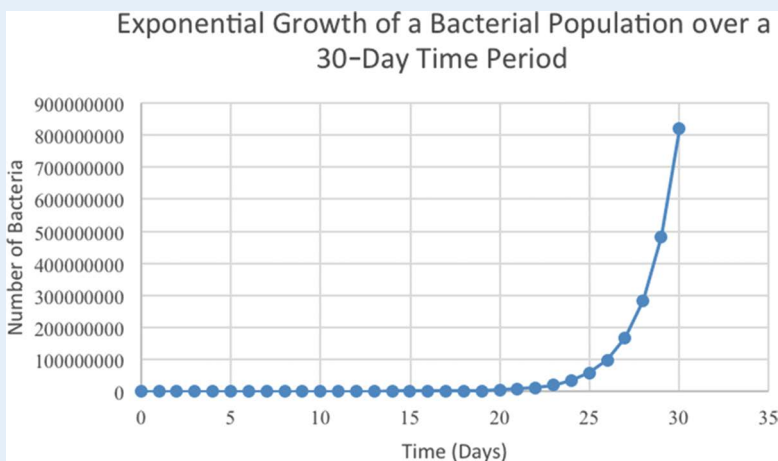
$$\frac{\Delta N}{\Delta t} = rN$$

$$\frac{\Delta N}{\Delta t} = (0.7)(170)$$

$$\frac{\Delta N}{\Delta t} = 119$$

If we repeat this calculation until day 10, we will see the following for our population growth:

Day	Number of Bacteria
0	100
1	170
2	289
3	491
4	835
5	1420
6	2414
7	4103
8	6976
9	11859
10	20160

If we continued this for the next 30 days, the growth of our population would look like this:

Exponential Growth of a Bacterial Population over a 30-Day Time Period

Logistic growth example

Say that the same population of bacteria from the previous example is experiencing logistic growth at the same rate, but the carrying capacity of the environment is 2000 bacteria. Let's calculate the growth rate of the population under these circumstances:

$$\frac{\Delta N}{\Delta t} = r_{max} N \frac{(K-N)}{K}$$

$$\frac{\Delta N}{\Delta t} = (0.7)(100)\left[\frac{(2000-100)}{2000}\right]$$

$$\frac{\Delta N}{\Delta t} = (70)(0.95)$$

$$\frac{\Delta N}{\Delta t} = 66.5$$

The growth rate is slower under these circumstances due to the limited availability of resources.

If you need more information, the following tutorial can help to further explain these concepts:

Khan Academy: Exponential and logistic growth in populations

Calculation of Doubling Time

When a population is experiencing exponential growth, the time it takes for the population to double, called doubling time, can be calculated using the **rule of 70**, which is expressed as

$$t = \frac{70}{r}$$

where t is doubling time in years, and r is the growth rate expressed as a percentage. For example, if a population is growing exponentially at a rate of 14% per year, it will take five years for the population to double in size:

$$t = \frac{70}{r}$$

$$t = \frac{70}{14}$$

$$t = 5 \, years$$

Calculation of Population Growth Rate

This equation can also be used to calculate the rate at which a population is growing. For instance, if a population in exponential growth doubles its population in eight years, its rate of growth is

$$t = \frac{70}{r}$$

$$8 = \frac{70}{r}$$

$$8r = 70$$

$$r = \frac{70}{8}$$

$$r = 8.75\%$$

If you need more information, the following tutorial can help to further explain these concepts:

Khan Academy: Exponential and logistic growth

Interpretation of Age-Structure Diagrams

Age-structure diagrams tell us how fast a given population is growing; for example, the population of a state, province, or country. These diagrams are generated by plotting the percentage of males and females in three categories of ages, or cohorts: prereproductive (0–14), reproductive (15–44), and postreproductive (45–85+). Individuals of the population in the prereproductive years and reproductive years have the potential to have children, which will increase the size of the population. Individuals in the postreproductive years are no longer having children and are therefore not contributing to the growth of the population.

For example, in figure 5.2, Country X has the largest rate of population growth. This is because there is a greater percentage of the population in the prereproductive and reproductive years than in the postreproductive years. Country Y shows the smallest rate of population growth because the percentage of the population in each of the three cohorts is about equal in size.

We can also see that Country X has a higher infant mortality rate than Country Y or Country Z, because the youngest members of the population are not surviving to adulthood. From this, we can infer that Country X is probably an unindustrialized or developing country (LDC), which usually has a higher infant mortality rate, while Country Y and Country Z probably are industrialized or developed countries (MDC), which usually have a lower infant mortality rate. This is typically due to uncontaminated drinking water, better nutrition, better healthcare, and the education of women.
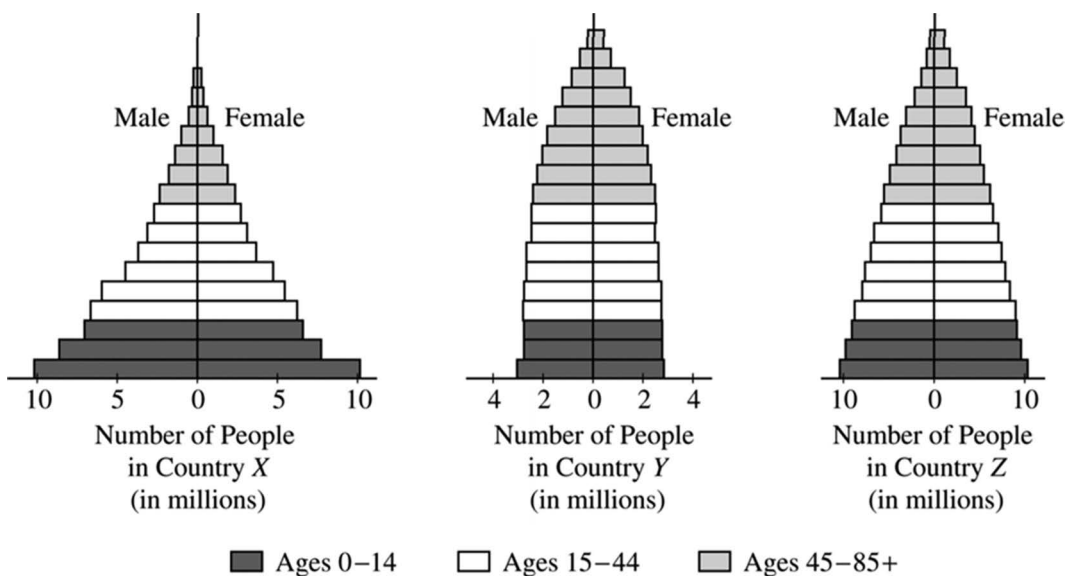
Figure 5.2 Age Structure Diagrams for Hypothetical Countries X, Y, and Z

If you need more information, the following tutorial can help to further explain these concepts:

Khan Academy: Life tables, survivorship, & age-sex structure

Energy Resources and Consumption

Determination of Half-Life

The **half-life** of a substance is the time it takes for 50% of the substance to decay, and is represented by the following formula:

$$N(t) = N_o \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

where $N(t)$ is the amount of substance at time t, N_0 is the initial amount of the substance, and $t_{1/2}$ is the half-life of the substance in years.

Example

After 100 million years, only 1/32 of the original amount of a particular radioactive waste will remain. The half-life of this radioactive waste is how many million years?

$$N(t) = N_o \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$\frac{N(t)}{No} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$\log_{1/2}\left(\frac{N(t)}{N_o}\right) = \frac{t}{t_{1/2}}$$

$$t_{1/2} = \frac{t}{\log_{1/2}\left(\frac{N(t)}{N_o}\right)}$$

$$t_{1/2} = \frac{100}{\log_{1/2}\left(\frac{1}{32}\right)} = 5$$

Pollution

Interpretation of Dose-Response Curves

A dose-response curve shows the effect of a toxic chemical on an organism or population. The concentration of the substance is its dose and the effect is the response. This relationship is typically shown in a graph like figure 5.3, which shows the effect of $CuSO_4$ on a population of brine shrimp.
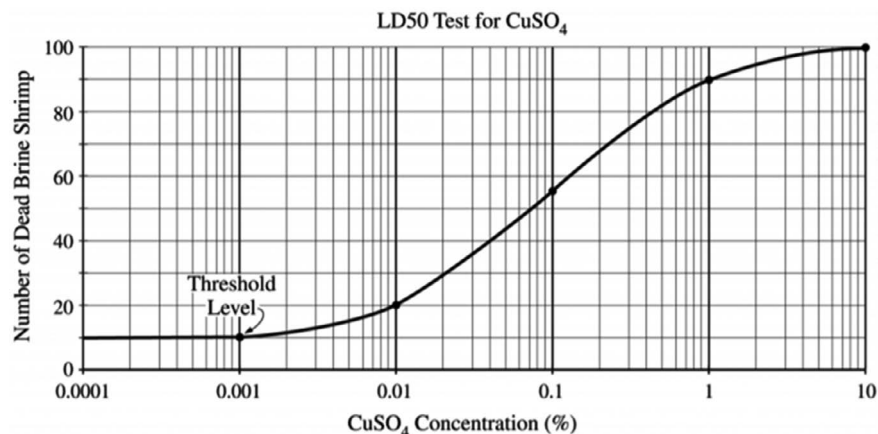

Figure 5.3 LD$_{50}$ Test for CuSO$_4$

From this graph we can determine the threshold level of toxicity of $CuSO_4$ as well as its LD$_{50}$. The **threshold level of toxicity** is the dose below which no toxic (or lethal) effects are observed and/or above which toxic (or lethal) effects are apparent. In figure 5.4, 0.001% $CuSO_4$ is the threshold level of toxicity because there is an observable increase in the death of brine shrimp after that concentration.

The LD_{50} is the **lethal dose** of a toxic substance or chemical that kills 50% of the test population (test organisms/specimens). Similar to the LD_{50} is the ED_{50}, which is the amount or dose of a toxic substance or that causes an observable or undesirable effect or desired symptom in 50% of the test population (test organisms/specimens) without killing them.

In figure 5.3 the 10 brine shrimp that are already dead at even the lowest concentration of $CuSO_4$ are the control group; this is natural mortality based on the threshold level of toxicity. So, the test population is 90 brine shrimp and 50% of this test population is 45 brine shrimp. Therefore, according to the data plotted on the graph, the LD_{50} for brine shrimp in a $CuSO_{44}$ solution is at a $CuSO_4$ concentration of 0.1%. This corresponds to the deaths of the control population (10 shrimp) plus the test population (45 shrimp), which equals 55 dead brine shrimp.